Nonlinear, Linear Analysis and Computer-Aided Design of Resistive Mixers

SHUNICHIRO EGAMI, MEMBER, IEEE

Abstract—**Nonlinear large-signal analysis of local current shape and linear small-signal analysis of small-signal products are made for resistive mixers. Iteration adapted from Newton's method was used in the nonlinear analysis. The conjugate match method was used in the linear analysis to find the minimum conversion loss. These theories were applied to a new type spurious suppressed mixer and a reliable computer simulation was made.**

I. Introduction

WITH THE ADVENT of the Schottky barrier diode, performance of the mixer was substantially improved. At the same time, mathematical treatment of the mixer was also made by many investigators [1]–[5]. In spite of the existence of detailed mixer theories, actual mixer circuit design still involves a laborious trial and error method, and realistic realizable minimum conversion loss cannot be found theoretically, even if diode characteristics are correctly given. The difficulty in applying existing mixer theory to the actual circuit design arises from the fact that the amplitude and phase of the local and its harmonic current, which is a fundamental determinant of mixer performance, cannot be obtained for an actual mixer in general, and that the necessary order of harmonics to make a realistic mixer analysis is not clearly given. The latter difficulty arises because, although the power of the nth local harmonic decreases by $1/n^2$ [6], amplitude of the nth harmonic current has no such restriction. Therefore, there is no sure reason to consider the first two harmonics only, as was always done in previous papers.

The aim of this paper is to provide a more realistic theoretical estimate of the mixer performance, by which actual circuit design can be made. The local and its harmonic current are obtained, when load admittances to each harmonic are given, which was not considered in previous papers. The conversion matrix, which is determined by the harmonic current, is used to get an optimum mixer performance. These nonlinear and linear analyses are combined into a single computer program, in which the order of the harmonics considered can be increased as desired. These analyses are applied to a new type mixer in which leakage of unnecessary harmonic and small-signal product are suppressed, thus making a low conversion loss possible.

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The author is with the Electrical Communication Laboratories, Nippon Telegraph and Telephone Public Corporation, Yokosuka-shi, 238-03, Japan.

II. Nonlinear Large-Signal Analysis to Find the Elements of the Conversion Matrix

When local oscillator power with frequency ω_p is fed into the diode, currents with frequency $n\omega_p$ $(n = 0,1,2,\cdots)$ flow across the junction of the diode. These currents determine the small-signal admittance of the diode. Therefore, the conversion matrix which determines the relation among small-signals is determined by these currents. Since finding these large-signal currents requires a nonlinear analysis, it is impossible to get a general analytical solution. However, using the iteration method, a solution can be found, if the load admittances to each harmonic are prescribed.

At the junction of the diode, voltage V and current I satisfy the following relation:

$$I = I_s \exp(\beta V) \tag{1}$$

where I_s and β are determined by the diode used.

Actually, the diode has a series resistance R_s, junction capacitance C_j, and lead inductance L_s, but these elements can be included in the load admittances which represent the outer circuit seen from the junction. Differentiating (1) by V, one obtains the small-signal admittance G_j:

$$G_j = \beta I. \tag{2}$$

This equation indicates that the small-signal admittance is determined by the current of the diode.

When local oscillator power is fed into the diode, harmonic $(n\omega_p,\ n = 0,1,2,\cdots)$ currents and voltages arise in the diode. Therefore, V and I can be represented as follows, if the first N harmonics are taken into consideration:

$$V = \sum_{n=-N}^{N} V_n \exp(jnx) \tag{3}$$

$$I = \sum_{n=-N}^{N} I_n \exp(jnx) \tag{4}$$

where

$$x = \omega_p t \qquad V_n = V_{-n}^* \qquad I_n = I_{-n}^*.$$

The equivalent circuit for this large-signal nonlinear analysis is shown in Fig. 1. Y_n is the load admittance to the nth harmonic $(n\omega_p)$ generated in the diode, and E_p is the voltage of the local oscillator. Since V and I of (3) and (4) satisfy (1), the mth harmonic current I_m, for example, can be determined by V_n $(n = 0,1,2,\cdots N)$, from the following equation:

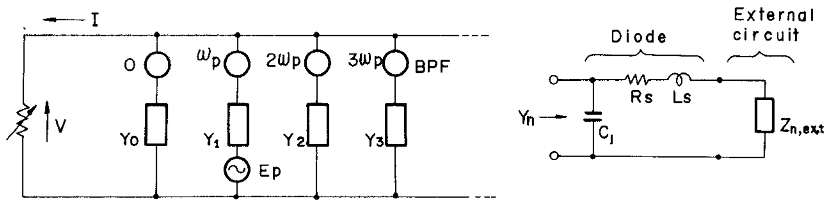

Fig. 1. Equivalent circuit for nonlinear analysis, which determines the shape of the local current. Y_n: load admittance to the nth harmonic, $Z_{n,\mathrm{ext}}$: External circuit impedance at the nth harmonic.

$$I_m = \frac{1}{2\pi}\int_{-\pi}^{\pi} I_s \exp\{\beta \sum_{n=-N}^{N} V_n \exp(jnx) - jmx\}\, dx. \quad (5)$$

Besides, each harmonic satisfies the following equations, as understood by Fig. 1:

$$I_k = -Y_k\{V_k - E_p\cdot\delta(k-1)\}, \qquad k = 0,1,2,\cdots,N \quad (6)$$

where δ means the delta function, $\delta(0) = 1$, otherwise $= 0$.

Here, these harmonic voltages and currents and the load admittances are represented in matrix form as

$$\boldsymbol{V} = \begin{pmatrix} V_0 \\ V_1 \\ \vdots \\ V_N \end{pmatrix}, \quad \boldsymbol{E}_p = \begin{pmatrix} 0 \\ E_p \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \boldsymbol{I} = \begin{pmatrix} I_0 \\ I_1 \\ \vdots \\ I_N \end{pmatrix},$$

$$Y = \begin{pmatrix} Y_0 & & & 0 \\ & Y_1 & & \\ & & \ddots & \\ 0 & & & Y_N \end{pmatrix}. \quad (7)$$

Using these relations, (6) can be expressed as follows:

$$\boldsymbol{I} + Y\cdot(\boldsymbol{V} - \boldsymbol{E}_p) = 0. \quad (8)$$

Since the current vector $\boldsymbol{I}$ is a function of the voltage vector, $\boldsymbol{V}$ as understood from (5), the above equation can be considered as a nonlinear equation for $\boldsymbol{V}$. This nonlinear equation is solved by an iteration method adapted from Newton's method [10].

We represent the left-hand side of (8) by $\boldsymbol{F}(\boldsymbol{V})$, as

$$\boldsymbol{F}(\boldsymbol{V}) = \boldsymbol{I}(\boldsymbol{V}) + Y\cdot(\boldsymbol{V} - \boldsymbol{E}_p) \quad (9)$$

where the kth element of this vector is given as follows:

$$F_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} I_s \exp\{\beta \sum_{n=-N}^{N} V_n \exp(jnx) - jkx\}\, dx + Y_k\{V_k - E_p\cdot\delta(k-1)\}. \quad (10)$$

Then (8) is written as

$$\boldsymbol{F}(\boldsymbol{V}) = 0. \quad (11)$$

If we represent the 0th order approximate solution of (11) by $\boldsymbol{V}^{(0)}$ (this is the initial value of the iteration), correction vector $\delta\boldsymbol{V}$ is determined by the following equation:

$$\boldsymbol{F}(\boldsymbol{V}^{(0)}) + D\cdot\delta\boldsymbol{V} = 0 \quad (12)$$

where vectors $\boldsymbol{F}$ and $\delta\boldsymbol{V}$ and matrix D are defined as follows:

$$\boldsymbol{F} = \begin{pmatrix} F_0 \\ F_1 \\ \vdots \\ F_N \end{pmatrix}, \quad \delta\boldsymbol{V} = \begin{pmatrix} \delta V_0 \\ \delta V_1 \\ \vdots \\ \delta V_N \end{pmatrix}$$

$$D = \begin{pmatrix} \frac{\partial}{\partial V_0}F_0 & \frac{\partial}{\partial V_1}F_0 & \cdots & \frac{\partial}{\partial V_N}F_0 \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial}{\partial V_0}F_N & \frac{\partial}{\partial V_1}F_N & \cdots & \frac{\partial}{\partial V_N}F_N \end{pmatrix}. \quad (13)$$

The element of matrix D, for example, $(\partial/\partial V_m)F_k$ is given by the differentiation of (10) as

$$\frac{\partial}{\partial V_m}F_k = \frac{\beta}{2\pi}\int_{-\pi}^{\pi} I_s \exp\{\beta \sum_{n=-N}^{N} V_n \exp(jnx) - j(k-m)x\}\, dx + Y_k\cdot\delta(k-m) = \beta I_{k-m} + Y_k\cdot\delta(k-m). \quad (14)$$

Therefore, matrix D can be written as

$$D = \beta \begin{pmatrix} I_0 & I_{-1} & \cdots & I_{-N} \\ I_1 & I_0 & \cdots & I_{-N+1} \\ \cdots & \cdots & \cdots & \cdots \\ I_N & I_{N-1} & & I_0 \end{pmatrix} + \begin{pmatrix} Y_0 & & & 0 \\ & Y_1 & & \\ & & \ddots & \\ 0 & & & Y_N \end{pmatrix}. \quad (15)$$

Since the elements of this matrix I_n are given by (5) for $\boldsymbol{V} = \boldsymbol{V}^{(0)}$, correction vector $\delta\boldsymbol{V}$ can be calculated by the following equation:

$$\delta\boldsymbol{V} = -D^{-1}\cdot\boldsymbol{F}(\boldsymbol{V}^{(0)}). \quad (16)$$

Using this correction vector, the first-order approximate solution $\boldsymbol{V}^{(1)}$ is given as

$$\boldsymbol{V}^{(1)} = \boldsymbol{V}^{(0)} + \delta\boldsymbol{V}. \quad (17)$$

Repeating this procedure, if $\boldsymbol{V}^{(n)}$ converges, the converged vector can be considered as the solution of (11). Convergence of this iteration depends on the initial vector

$V^{(0)}$. Although the author could not specify the boundary of convergence for $V^{(0)}$, intuitive selection of $V^{(0)}$ as

$$V^{(0)} = \boldsymbol{E}_p \tag{18}$$

was always successful. This is credible, because, when all harmonics are short circuited, solution vector $\boldsymbol{V}$ is equal to $\boldsymbol{E}_p$. Application of this method to the practical problem will be described in Section IV.

III. Linear Analysis to Find Minimum Conversion Loss

Frequency conversion of the input signal is carried out by the modulated admittance of the diode. Since input signal power is very small compared with the local oscillator power, the relation among small-signal products generated by the input signal can be considered linear. Admittance of the diode, which is modulated by the local oscillator power, can be expressed in accordance with the former section as

$$G_j = \sum_{n=-N}^{N} g_n \exp\,(jn\omega_p t) \tag{19}$$

where

$$g_n = g_{-n}^*.$$

g_n can be related to the harmonic current of the diode using (2) and (4) as

$$g_n = \beta I_n, \qquad n = 0,1,2,\cdots N. \tag{20}$$

When an input signal is fed into this modulated admittance, small-signal products are generated. Here these small-signal voltages and currents are represented as [9]

$$v = \sum_{n=-N}^{N} v_n \exp\,[j(n\omega_p + \omega_{\mathrm{IF}})t] \tag{21}$$

$$i = \sum_{n=-N}^{N} i_n \exp\,[j(n\omega_p + \omega_{\mathrm{IF}})t] \tag{22}$$

where ω_{IF} is the output IF frequency. These small-signal voltages and currents and the conversion matrix are conveniently represented in matrix form as follows:

$$\boldsymbol{v} = \begin{pmatrix} v_{-N} \\ \vdots \\ v_0 \\ \vdots \\ v_N \end{pmatrix}, \quad \mathring{\boldsymbol{e}} = \begin{pmatrix} i_{-N} \\ \vdots \\ i_0 \\ \vdots \\ i_N \end{pmatrix}$$

$$G = \begin{pmatrix} g_0 & g_{-1} & \cdots & g_{-N} \\ g_1 & g_0 & \cdots & g_{-N+1} \\ \cdots & \cdots & \cdots & \cdots \\ g_N & g_{N-1} & \cdots & g_0 \end{pmatrix}. \tag{23}$$

Using these representations, the following relation is obtained between the small-signal voltages and currents:

$$\mathring{\boldsymbol{e}} = G \cdot \boldsymbol{v}. \tag{24}$$

This equation represents the relation across the nonlinear resistance of the diode. The parasitic elements of the diode R_s,C_j,L_s can be included in the equivalent circuit as shown in Fig. 2.

By defining the matrices P and Q as

$$P = \begin{pmatrix} j(-N\omega_p + \omega_{\mathrm{IF}})C_j & & & & & & 0 \\ & \cdot & & & & & \\ & & \cdot & & & & \\ & & & j\omega_{\mathrm{IF}}C_j & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ 0 & & & & & & j(N\omega_p + \omega_{\mathrm{IF}})C_j \end{pmatrix}$$

$$Q = \begin{pmatrix} R_s + j(-N\omega_p + \omega_{\mathrm{IF}})L_s & & & & & & 0 \\ & \cdot & & & & & \\ & & \cdot & & & & \\ & & & R_s + j\omega_{\mathrm{IF}}L_s & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ 0 & & & & & & R_s + j(N\omega_p + \omega_{\mathrm{IF}})L_s \end{pmatrix} \tag{25}$$

the modified conversion matrix G', which includes the effects of R_s,C_j,L_s is given as follows:

$$G' = (1 + PQ + GQ)^{-1} \cdot (P + G). \tag{26}$$

If load admittances to the small-signal products other than input signal and output IF are given, from the conversion matrix G', one can deduce a two-port matrix which relates the input signal to output IF. If the load

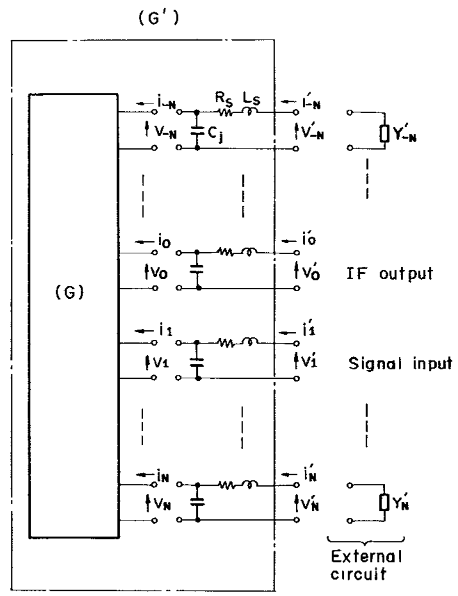

Fig. 2. Equivalent circuit for the small-signal products. Matrix G represents the conversion at the junction. Matrix G' includes the effects of R_s, C_j, and L_s.

admittance to the small-signal product $k\omega_p + \omega_{IF}$ is represented by Y_k' as shown in Fig. 2, and matrix Y' is defined as

$$Y' = \begin{pmatrix} Y'_{-N} & & & & & & & \\ & \ddots & & & & 0 & & \\ & & Y'_{-1} & & & & & \\ & & & 0 & & & & \\ & & & & 0 & & & \\ & & & & & Y'_{2} & & \\ & 0 & & & & & \ddots & \\ & & & & & & & Y'_{N} \end{pmatrix} \begin{matrix} \\ \\ \cdots \mathrm{IF} \\ \\ \cdots \mathrm{signal} \\ \\ \\ \\ \end{matrix} \tag{27}$$

then, conversion to a two-port matrix is made by matrix inversion as

$$Z = (G' + Y')^{-1}. \tag{28}$$

Taking out the elements of this matrix, a two-port matrix can be made as follows:

$$\begin{pmatrix} v_0' \\ v_1' \end{pmatrix} = \begin{pmatrix} Z_{00} & Z_{01} \\ Z_{10} & Z_{11} \end{pmatrix} \begin{pmatrix} i_0' \\ i_1' \end{pmatrix}. \tag{29}$$

This impedance matrix can be transformed to a transmission matrix [11] by the following equation:

$$A = Z_{11}/Z_{01} \quad B = -Z_{10} + Z_{11} \cdot Z_{00}/Z_{01}$$
$$C = 1/Z_{01} \quad D = Z_{00}/Z_{01}. \tag{30}$$

If these parameters are real, signal-IF conversion loss takes a minimum value when the signal and IF port are matched by the image impedances. This can be extended to the network with complex A,B,C,D by the conjugate matching method [5]. Conjugate match means that impedances of the network, seen from outside, are equal to the complex conjugate of the load impedances, and the conjugate matched signal and IF impedances can be considered as the optimum impedances which give the minimum conversion loss. Thus optimum impedance to the signal input is given as

$$Z_{\mathrm{signal}} = -\alpha_1 + (\alpha_1^2 + \alpha_2)^{1/2} \tag{31}$$

where

$$\alpha_1 = j\frac{\mathrm{Im}\,(B^*C + A^*D)}{2\,\mathrm{Re}\,(CD^*)} \qquad \alpha_2 = \frac{\mathrm{Re}\,(AB^*)}{\mathrm{Re}\,(CD^*)} \tag{32}$$

and optimum impedance to the IF output is given as

$$Z_{\mathrm{IF}} = -\gamma_1 + (\gamma_1^2 + \gamma_2)^{1/2} \tag{33}$$

where

$$\gamma_1 = j\frac{\mathrm{Im}\,(AD^* + B^*C)}{2\,\mathrm{Re}\,(AC^*)} \qquad \gamma_2 = \frac{\mathrm{Re}\,(BD^*)}{\mathrm{Re}\,(AC^*)} \tag{34}$$

and conjugate matched conversion loss L_c is given as

$$L_c = \frac{\mathrm{Re}\,Z_{\mathrm{signal}}}{\mathrm{Re}\,Z_{\mathrm{IF}}} \cdot \frac{|AZ_{\mathrm{IF}}^* + B|^2}{|Z_{\mathrm{signal}}|^2}. \quad (3$$

These are the inherent values of the two-port matı Thus we can find the minimum conversion loss and oį mum signal IF impedances if the conversion matrix reduced to a two-port matrix.

IV. Spurious Suppressed Mixer—A Model of Computer Simulation

By the modulated admittance of the diode, the inį signal is converted to many frequencies. In these frequ cies, only the IF frequency power is derived as the outp Though other frequencies are not necessary to the mi: operation, termination impedances at these frequenc considerably affect the mixer performance. Since th unnecessary frequency products have a higher frequen they can transmit in the waveguide by a higher or mode. Therefore, it is difficult to suppress the leakage these powers by a signal bandpass filter (BPF). Leak of these small-signal powers to the outer circuit degra the conversion loss and the delay time characteristics the mixer. To prevent this, usually a waffle-iron type lo pass filter (LPF), which has no spurious response o several multiples of its passband frequency, was used [The characteristics of the waffle-iron type LPF can realized by the coaxial LPF, which approximates t lumped constant LPF. Use of the latter type LF for rejection of unwanted harmonics and small-sig products, is preferable for the following reasons. 1) U wanted small-signals and harmonic are reflected near t diode, and leakage to IF and signal ports are prevent simultaneously. Also, ambiguity to the terminations these frequencies is greatly reduced. 2) It is easy to desi especially in the upper microwave and millimeter-wa band.

Fig. 3(a) shows the 20-GHz mixer using the latter ty coaxial LPF. LPF 1 is the 20-GHz LPF which p $\omega_{IF}, \omega_p \pm \omega_{IF}$ and reject $2\omega_p, 2\omega_p \pm \omega_{IF} \cdots$. A lumped c stant LPF, which is designed by an image impedar method, was approximated by the coaxial line (capacitar by low impedance line, inductance by high impedar line). Fig. 3(b) shows the computer simulated transm sion characteristic of the designed coaxial LPF. Thus can be understood that by this type of coaxial LPF, jection is possible to over several multiples of the passba frequency.

Hereafter the characteristics of this mixer are compu simulated by the method described in Sections II and I Local frequency and IF frequency are chosen as 20 a 0.5 GHz, respectively. The typical diode used in this f quency band is a $10\mu\phi$ GaAs planar Schottky diode, whi has the series resistance $R_s = 3\ \Omega$, lead inductance L_s 0.3 nH, junction capacitance $C_j = 0.15$ pF, saturati current $I_s = 10^{-13}$ A (in Fig. 6, $I_s = 10^{-9}$ A is also c sidered), and $\beta = 30\ \mathrm{V}^{-1}$.

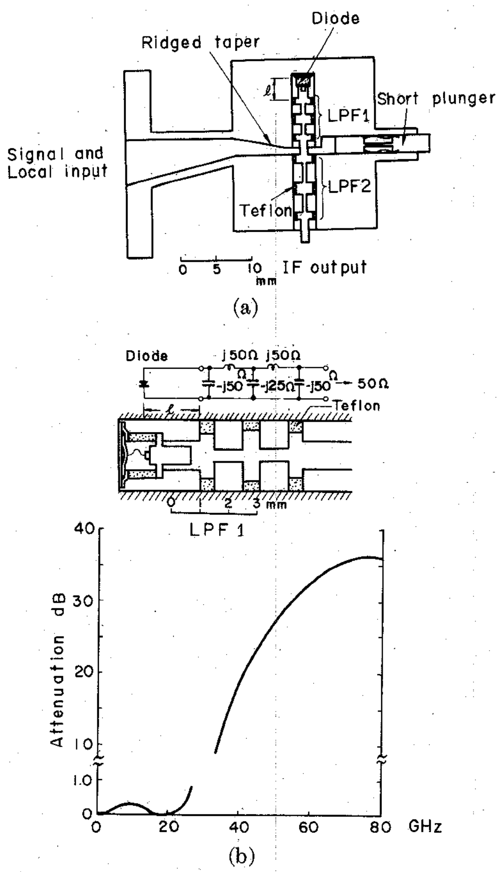

Fig. 3. (a) Spurious suppressed 20-GHz mixer which is used as a model of the computer simulation. (b) Design and characteristics of LPF 1.

Load admittances to the harmonic and small-signal products are changed by the length l (shown in Fig. 3) between diode and LPF 1. On the other hand, image frequency power can pass LPF 1, and is reflected at the signal BPF. Therefore, load admittance to the image frequency does not depend on l, and can be determined independently, changing the line length to the signal BPF. Fig. 4 shows the local and its harmonic current, determined by the method stated in Section II as a function of line length l. In the computation, up to the third local harmonic (which means $N = 3$) was considered. Iteration was converged by 30 $\sim$ 50 steps for reasonable local power. More steps were necessary for comparatively large local power input. Conjugate matched conversion loss and signal IF impedances, which correspond to the complex local current were computed and are also shown in the figure. Image termination was optimized independently to give a lowest conversion loss.

Fig. 5 shows the effect of image termination. Image termination can be changed by line length L between diode and signal BPF. $L/\lambda_{\text{image}} = 0$ and 0.5 correspond to image short, and $L/\lambda_{\text{image}} = 0.25$ to image open. Optimum image termination is neither short nor open, if the actual mixer as shown here is considered. Fig. 6 shows the effect of the local oscillator power to conjugate matched conversion loss and optimum signal IF impedances. Saturation current of 10^{-13} A corresponds to the GaAs diode, and of 10^{-9} A corresponds to the silicon diode. Lower values of saturation current give a lower conversion loss, as expected. The result of the preliminary experiment is shown in Fig. 7. Line length l between diode and LPF 1, and image termi-

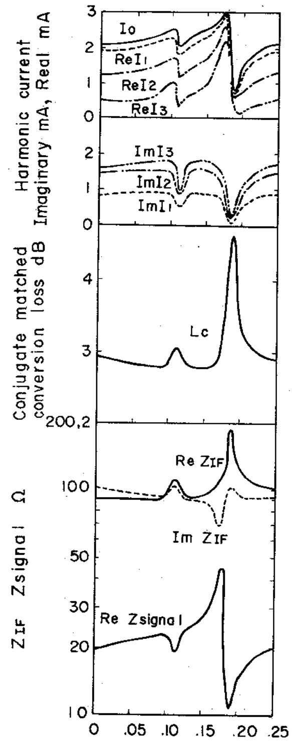

Fig. 4. Local current, conjugate matched conversion loss and signal, IF impedances are obtained as functions of l/λ_p. λ_p: wavelength of local input. Im Z_{signal} is lower than 10 Ω. Image termination is optimized at any point of l. $E_p = 1.1$ V.

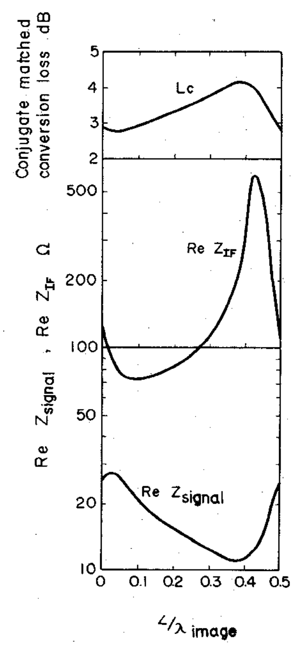

Fig. 5. Conjugate matched conversion loss and signal IF impedances as functions of image termination. L: line length between diode and signal BPF. λ_{image} image frequency wavelength. $l/\lambda_p = 0.125$; $E_p = 1.1$ V.

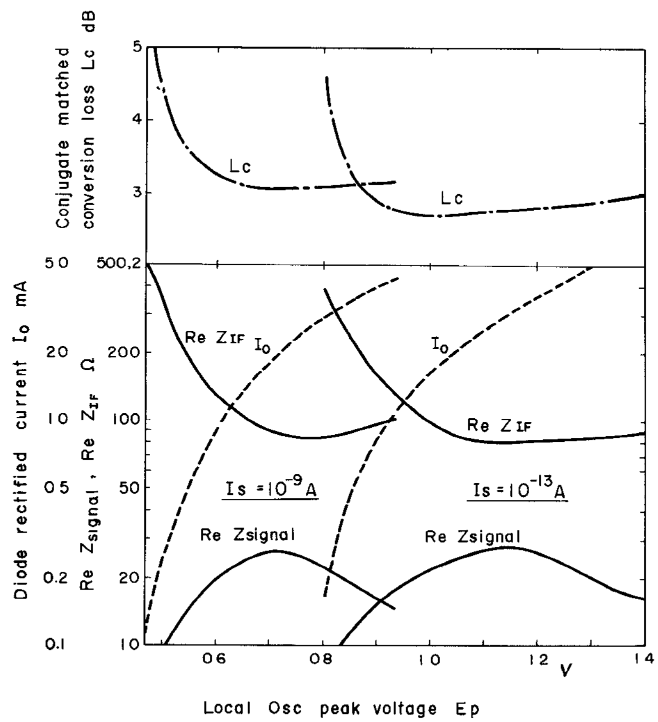

Fig. 6. Conjugate matched conversion loss and signal IF impedances as functions of local oscillator peak voltage. Image termination and l are optimized.

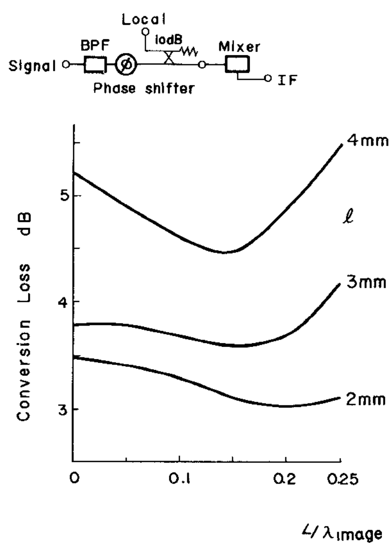

Fig. 7. Experimental conversion loss of the 20-GHz mixer shown in Fig. 2. Image termination and l are changed.

nation are changed for a constant rectified current of 5 mA. This experimental conversion loss may include the mismatch loss in the signal or IF port. Therefore, this result cannot be directly compared with the computer-simulated conversion loss of Fig. 5. However, it shows the advantage of the mixer shown here, compared with the conventional mixer.

V. Conclusion

Large-signal nonlinear analysis of the local current and small-signal linear analysis of the small-signal products were made. The first analysis, which has been rather bypassed heretofore, is solved using the iteration method. The correct solution of the first analysis made the solution of the second analysis reliable.

Since the conversion matrix was complex, the conjugate matching method was used to get a minimum conversion loss. These analyses were applied to the spurious suppressed mixer. Results of the simulation showed the feasibility of the method.

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